Indian Statistical Institute
Midterm Examination
B.Math II Year
Numerical Method

Time: 3 hours | Date: March 9, 2007

Answer all questions. They carry equal marks.

(1) What is the relationship between x_{n+1} and x_n in the application of Newton's method to the equation $f(x) = 0$? What formula does one get when $f(x) = x^2 - c$?

(2) Derive a recursion formula for calculating

$$y_n = \int_0^1 \frac{x^n}{4x+1}\, dx\,.$$

Give an algorithm that works well and another that doesn't – both based on the same recursion formula!

(3) We wish to compute $(\sqrt{2}-1)^6$ using the approximate value 1.4 for $\sqrt{2}$. You may then choose to substitute this approximate value in the original expression or in any one of the following equivalent expressions:

$$\frac{1}{(\sqrt{2}+1)^6},\ (3-2\sqrt{2})^3,\ \frac{1}{(3+2\sqrt{2})^3},\ 99-70\sqrt{2},\ \frac{1}{99+70\sqrt{2}}.$$

Which alternative gives the best result?

(4) Use the series $\frac{1}{2}\ln\frac{1+y}{1-y} = y + \frac{y^3}{3} + \frac{y^5}{5} + \cdots$ to compute $\ln 1.2$ with an error less than 10^{-7}. How many terms would be needed if the expression for $\ln(1+y)$ with $y = 0.2$ were used instead?

(5) Compute to four significant digits: $\int_{10}^{\infty} \frac{1}{\sqrt{x^3+x}} dx$.

(6) The water level in north sea is mainly determined by the M_2 - tides whos e period is about 12 hours and thus has the form

$$H(t) = h_0 + a_1 \sin\frac{2\pi t}{12} + a_2 \cos\frac{2\pi t}{12},$$

t in hours. Suppose we have made the following observations:

t	0	2	4	6	8	10	hour
$H(t)$	1.0	1.6	1.4	0.6	0.2	0.8	meters

Fit $H(t)$ to the series of measurements using the method of least squares.

(7) Determine for $f(x) = \pi^2 - x^2$, the cosine polynomial $f^*(x) = \sum_{j=0}^{n} c_j \cos jx$ which makes $\|f^* - f\|_2$ on the interval $[o, \pi]$ as small as possible.